Footstep Illusion Art

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Abstract

This paper proposes the methodology of art using an optical illusion called the footsteps illusion. We consider the mechanisms of this illusion, determine the conditions for maximizing the strength of the illusion, classify the apparent motions into eight patterns according to the object widths of and the distance between objects, and create new illusion artworks by combining these eight patterns. Moreover, we demonstrate that we can create apparent rotation generated by pure translation. In the case of the footsteps illusion, the object is a rectangle; however, in the case of apparent rotation, the object is a set of four long thin rectangles forming a square. When the squares move in front of an orthogonal grid, they look as if they were rotating. Furthermore, we can control the direction of rotation; we can place two squares having identical shapes in such a way that they rotate in opposite directions.

Keywords: footsteps illusion, optical illusion artworks, computational illusion.

AMS 2010 Mathematics Subject Classification: 00A66 (Mathematics and Visual Art, Visualization)

1 Introduction

Illusion is a phenomenon that confuses the sensory functions. Consequently, research into illusions is nothing other than direct research into sensory functions. Furthermore, it is becoming generally accepted that illusion is useful in our daily lives rather than being an undesirable pathological phenomenon. The advantages of the use of mathematics are that we can describe the strength of an optical illusion quantitatively, estimate what will occur if we change some condition, and thereby determine how to increase or decrease the strength of the optical illusion. Moreover, we can create new optical illusion artworks by calculating the condition.

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Visual effects built on optical illusions have long been used in art [5, 16]. Escher [5] and Anno [1] created artworks depicting impossible scenes. Arcimboldo [8] drew hybrid images combining two different types of objects; for example, he represented faces with vegetables and fruits. Escher [5] and Fukuda [7] created artworks using reversible figures. Many other artworks also use optical illusion [6, 12, 17], but only persons who have particular empirical knowledge have been able to create optical illusion artworks hitherto. It would be useful if a methodology could be given so that anyone could create optical illusion artworks.

In this paper, we concentrate on a class of motion illusions, collectively named the footsteps illusion, discovered by Anstis [2]. In this illusion, translating objects look as if they were deforming when they pass through stripe patterns. We formulate the conditions under which the strength of the footsteps illusion [2] is maximized and propose a methodology for creating new illusion artworks based on the footsteps illusion. This paper is organized as follows. In Sec. 2, we review the footsteps illusion in detail and formulate the conditions for maximizing its strength. In Sec. 3, we classify apparent motion into eight patterns, which can be used for creating footsteps illusion artworks. In Sec. 4, we introduce applications of our methodology and our created optical illusion artworks. Finally, Sec. 5 is devoted to the concluding discussions.

2 'Footsteps illusion' and its strength

When black and white rectangles drift steadily across stationary dark-gray and light-gray stripes, the black and white rectangles appear to stop and start alternately, as shown in Fig. 1. This illusion is the 'footsteps illusion' [2].

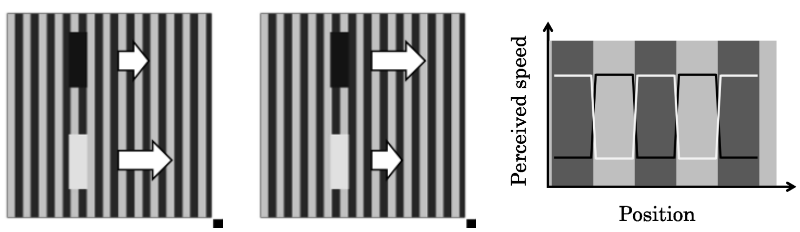


Figure 1: Footsteps illusion. The black (white) object appears to slow down when both vertical edges of the black (white) object move across a dark (light) gray stripe. The horizontal axis of the graph shows position of the object, and the vertical axis of the graph shows perceived speed.

The reason for this illusion is the change of the contrast between the object and the stripe [2–4, 10]. Miura et al. [13] proposed a mathematical model for explaining this illusion based on a time evolution equation of the extended FitzHugh–Nagumo system with two variables that correspond to the reaction and diffusion terms [13]. It has also been observed that the footsteps illusion is stronger in the peripheral visual field than in the central visual field [18].

This illusion occurs because humans have difficulty recognizing movement when the contrast is low between two (the leading and trailing; see Fig. 2) edges of the rectangle and the background. Therefore, the object appears to stop when the contrast is low between these two edges and the background, and the object appears to move when the contrast is high. We can control the strength of the illusion because the illusion occurs more strongly when the two edges are in the low contrast region for a longer time.

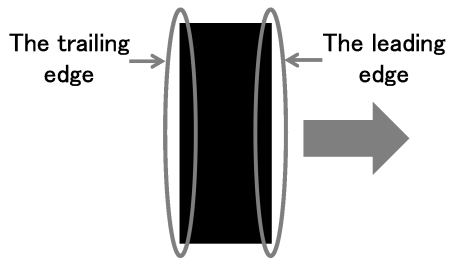


Figure 2: Leading and trailing edges of an object. The arrow indicates the direction of motion.

The strength of the illusion is quantified in the following way. Here, we define ω as the width of each stripe and x as the width of an object, as shown in Fig. 3. We assume that the object moves at a constant velocity. Let l_1 be the length of time during which both edges are within dark stripes, and l_2 be the length of time of one period (2ω) of the illusion. We put in place the condition $\omega \leq x$, which means that the object is longer than the stripe width. Let T be the ratio of l_1 to l_2.

a

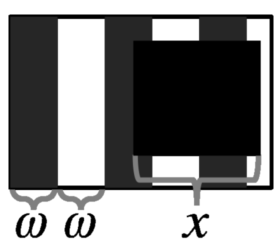


Figure 3: Definitions of ω and x.

Suppose that $\omega \leq x \leq 2\omega$. Then we can calculate the ratio T by

$$T = \frac{x - \omega}{2\omega}, \tag{1}$$

because the time length of one period is 2ω and the time length for both edges being with black stripes is $x - \omega$. Note that both time lengths are proportional to the velocity of the object. Therefore, T can be at most $1/2$, which is achieved when $x = 2\omega$, and T is at its minimum, 0, when $x = \omega$.

Next, suppose that $2\omega \leq x \leq 3\omega$. We calculate the ratio T in a manner similar to that above:

$$T = \frac{3\omega - x}{2\omega}. \tag{2}$$

As a result, in this case, T attains the maximum, $\frac{1}{2}$, when $x = 2\omega$ and the minimum, 0, when $x = 3\omega$.

Generalizing these observations, we can prove that T attains its maximum when $x = 2\omega, 4\omega, 6\omega \cdots$. This means that the footsteps illusion occurs most strongly when x is an even multiple of the stripe width. On the other hand, T attains its minimum when $x = \omega, 3\omega, 5\omega \cdots$. Therefore, the footsteps illusion occurs least strongly when x is an odd multiple of the stripe width.

We concluded above that the footsteps illusion occurs least strongly when x is an odd multiple of ω. Is this so? Actually, the object generates a different illusion in which the object looks as if it were changing width. This illusion is the 'inchworm illusion' [2].

In this way, we can generate footsteps and inchworm illusions by changing the width x.

3 Footstep illusion art

In this section, we propose a method for creating apparent motion using the footsteps illusion. The generated visual effect could be new art, which we call footstep illusion art.

In previous sections, we considered moving rectangles. Next, we consider an L-shaped object. As shown in Fig. 4, the width of the upper part of the object is 2ω and the width of the lower part is 3ω. When this L-shaped object moves horizontally across a stationary black and white stripe pattern with width ω, we can expect that the footsteps illusion occurs in the upper half and the inchworm illusion occurs in the lower half, which is what we can actually observe as the apparent motion. In this way, we can combine parts with even and odd multiples of the stripe width, and can create our desired motion.

Next we consider a square object that moves in a direction 45 degrees from horizontal, as shown in Fig. 5(a). Let the width of the stripe be ω, as before. If the object has width 2ω, then the object appears to move at a constant velocity in the vertical direction but to occasionally stop with respect to the horizontal direction. If instead the object has a width of 3ω, it still appears to move at a constant velocity in the vertical direction but instead appears to expand and shrink in the horizontal direction.

Next we replace the background with a square grid, as shown in Fig. 5(b). If we observe the apparent motion for the same two cases, in the case of $x = 2\omega$,

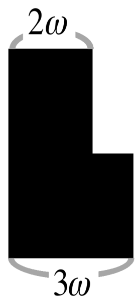
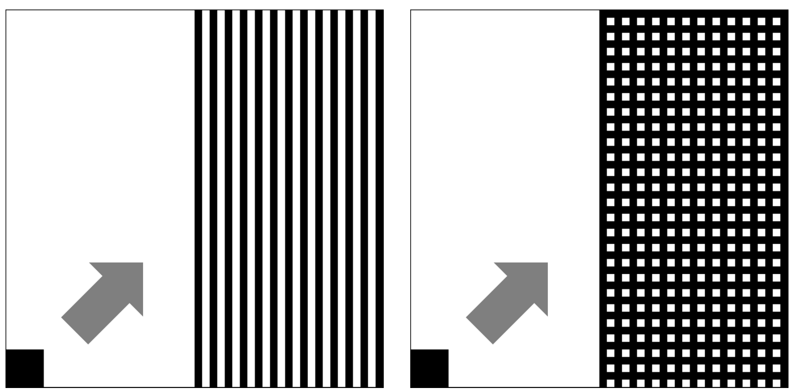


Figure 4: L-shaped object.



(a) (b)

Figure 5: Oblique motion.

the footsteps illusion occurs in both the vertical and horizontal directions, and in the case of $x = 3\omega$, the inchworm illusion occurs in both the vertical and horizontal directions. In this way, we can generate an optical illusion in both directions.

We have considered the width of an object thus far. Next we consider the relationship between the widths of two objects and the distance between them. Let the width of O_1 be x_1 and the width of O_2 be x_2. Now suppose that these two objects both move to the right, and let d be the distance between the leading edge of O_1 (the trailing object) and the trailing edge of O_2 (the leading object), as shown in Fig. 6.

If the horizontal positions of two objects overlap, we define d as the length of the overlapped portion, as shown in Fig. 7. We examine the visual effects that occur for all possible odd and even combinations of x_1, x_2, and d.

For example, when x_1, x_2, and d are all even multiples of the stripe width, footsteps illusions occur for O_1 and O_2 and their occurrence is synchronized.

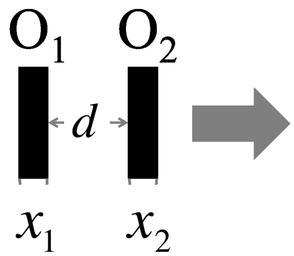


Figure 6: Widths of and distance between two objects. The arrow indicates the direction of motion.

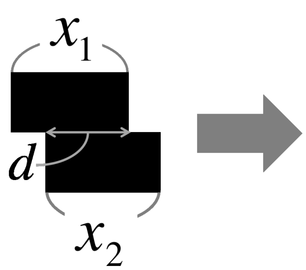


Figure 7: Case of two horizontally overlapped objects. The arrow indicates the direction of motion.

Examining all combinations of x_1, x_2, and d being even or odd multiples of the stripe width, we obtain eight patterns, which are summarized in Table 1.

In this table, the first column shows the pattern indexes assigned to the odd–even patterns of x_1, x_2, and d, which are given in the second, third, and fourth columns. The fifth and sixth column list the types of the illusions that O_1 and O_2 generate. Finally, the seventh column lists the timing relationship between the illusions of O_1 and O_2.

Table 1 shows a classification of illusion patterns generated by two objects. If we have three or more objects, we can anticipate the apparent motions on the basis of this table. We next introduce optical illusion artworks that combine many objects.

Artwork 1) "March"

Consider the figure consisting of the three objects a, b, and c shown in Fig. 8. The pattern of relative object widths and distances between objects is summarized in Table 2. In Table 2, the rows and columns correspond to objects, the diagonal cells correspond to the widths of the objects, and the off-diagonal cells correspond to distances between pairs of objects. The left lower part of

Table 1: Eight patterns of object widths and distance between objects.

	width			apparent motion		
	x_1	x_2	d	O_1	O_2	timing
1	even	even	even	footsteps	footsteps	synchronized
2	even	even	odd	footsteps	footsteps	alternating
3	odd	odd	even	inchworm	inchworm	alternating
4	odd	odd	odd	inchworm	inchworm	synchronized
5	even	odd	even	footsteps	inchworm	synchronized
6	even	odd	odd	footsteps	inchworm	alternating
7	odd	even	even	inchworm	footsteps	alternating
8	odd	even	odd	inchworm	footsteps	synchronized

this table is not shown; this is because the off-diagonal cells are symmetric with respect to the diagonal. Note that the values of the cells are not independent: some cells can be assigned arbitrarily, whereas the remaining redundant cell (value shown in parentheses) is strictly determined as a logical consequence. We use the same notation in the tables describing the other example illusion artworks.

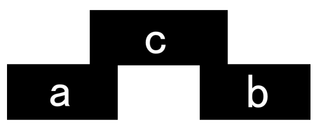


Figure 8: Worm.

Table 2: Pattern of widths of and distances between the objects in Fig. 8. 'even' ('odd') is even (odd) multiples of the stripe.

	a	b	c
a	even	odd	odd
b		even	(odd)
c			odd

When the figure in Fig. 8 moves to the right with a constant velocity in front of a stripe pattern, as shown in Fig. 9, the figure appears to change its shape like a worm.

Artwork 2) "For Egg Laying"

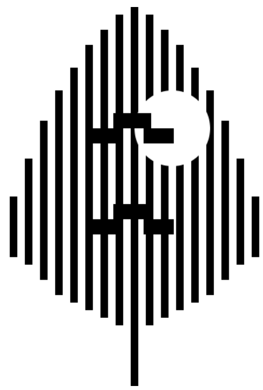


Figure 9: March.

In this artwork, the figure consists of the four objects a, b, c, and d and a central body, as shown in Fig. 10. Suppose that this figure moves to the right with a constant velocity in front of a stripe pattern in which the left half and the right half have different widths, as shown in Fig. 11. The pattern of relative widths and positions of objects is summarized in Tables 3 and 4 for the left and right sides of the stripe pattern, respectively.

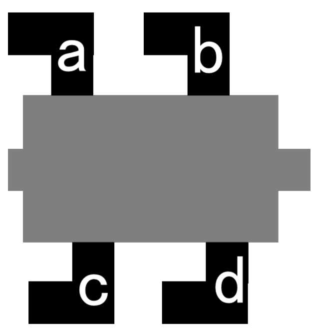


Figure 10: Turtle.

When the turtle shown in Fig. 10 moves in front of the stripe pattern shown in Fig. 11, the turtle appears to swim in the left half and to walk in the right half.

Artwork 3) "Bat under a Full Moon"

This figure consists of a central body and the two objects a and b, as shown in Fig. 12. Suppose that this figure moves 45 degrees from horizontal in front of a square grid, as shown in Fig. 13. The pattern of relative horizontal widths of and distance between the objects is shown in Table 5.

Table 3: Pattern of widths of and distances between the objects in Fig. 10 with respect to the left stripes of Fig. 11.

	a	b	c	d
a	even	even	even	even
b		even	(even)	(even)
c			even	(even)
d				even

Table 4: Pattern of widths of and distances between the objects in Fig. 10 with respect to the right stripes of Fig.11.

	a	b	c	d
a	even	odd	odd	even
b		even	(even)	(odd)
c			even	(odd)
d				even

When the bat shown in Fig. 12 moves in front of the grid pattern shown in Fig. 13, the bat appears to flutter its wings. We correctly perceive that the bat moves with constant velocity when it crosses in front of the full moon.

All of these artworks are posted at the first author's web page [15].

4 Apparent rotation generated by pure translation

For the two black square patterns shown in Fig. 14(a), when a square grid is translated from the upper left to the lower right behind the patterns, as indicated by Figs. 14(b) and (c), the patterns look as if they were rotating. Surprisingly, although the two square patterns have exactly the same shape,

Table 5: Pattern of horizontal widths of and distance between the objects in Fig. 12 with respect to the grid in Fig. 13.

	a	b
a	even	odd
b		even

Figure 11: For Egg Laying.

Figure 12: Bat.

they rotate in opposite directions: the upper pattern rotates clockwise, while the lower pattern rotates anticlockwise. This illusion is new in the sense that the rotation is generated by pure translation.

We can explain why translation generates rotation in the following way. The square pattern consists of four segments, the right, the bottom, the left, and the top. The grid is translated in the direction 45 degrees from horizontal and generates the inchworm illusion for each of the four segments separately over one grid period of the translation, resulting in the impression of rotation.

The reason why the patterns rotate in opposite directions can be understood in the following way. The relative location of a square pattern with respect to the grid determines the order in which the inchworm illusion occurs. The two square patterns are fixed in such a way that their relative positions differ by half a grid period, and consequently the inchworm illusion occurs for the second square in the opposite order as for the first square, i.e., right, top, left, and bottom rather than right, bottom, left, and top. Using this type of illusion, we can generate other types of artworks. The following are examples.

Artwork 4) "Drive"

The figure consists of the four objects a, b, c, and d shown in Fig. 15. Suppose that this figure moves to the right with constant velocity in front of a square grid, as shown in Fig. 16. The pattern of relative distances between

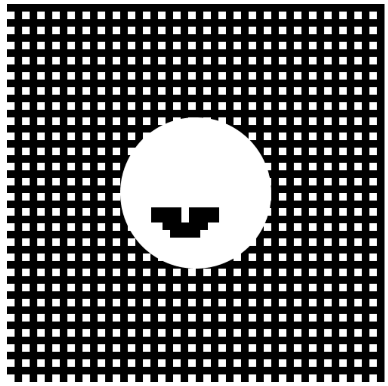
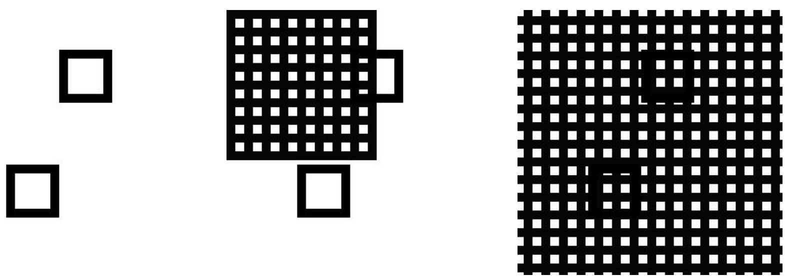


Figure 13: Bat under a Full Moon.



(a) (b) (c)

Figure 14: Apparent rotation generated by pure translation.

the objects is summarized in Tables 6 and 7 for pair a and c and pair b and d respectively.

When the wheels of automobile patterns, as shown in Fig. 15, move in front of a square grid, as shown in Fig. 16, the wheels appear to rotate.

Artwork 5) "Pinwheels"

The figure consists of the four objects a, b, c, and d shown in Fig. 17. Suppose that a square grid moves to the right with a constant velocity in front of these objects, as shown in Fig. 18. The pattern of relative distances between the objects is summarized in Tables 8 and 9.

When the square grid shown in Fig. 18 moves behind the figure shown in Fig. 17, the figure appears to rotate.

Artwork 6) "Eddy Current"

The figure consists of the objects a, b, c, and d shown in Fig. 19. Suppose that a square grid moves in the direction 45 degrees from horizontal with a constant velocity behind the figure, as shown in Fig. 20. The pattern of relative

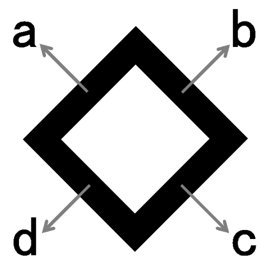


Figure 15: Wheel.

Table 6: Pattern of the widths of and distance between objects a and c in Fig. 15 with respect to the grid in Fig. 16.

	a	c
a	odd	even
c		odd

distances between the objects is summarized in Tables 10 and 11.

When the square grid shown in Fig. 20 moves behind a figure as shown in Fig. 19, the figure appears to rotate.

5 Practical applications

The footsteps illusion can be applied to not only optical illusion artworks but also other creations. We introduce two examples.

5.1 Design of a clock

Consider a sectoral object and stripe pattern as shown in Fig. 21. The sectoral object is attached to the seconds hand so that it rotates around the center of

Table 7: Pattern of the widths of and distance between objects b and d in Fig. 15 with respect to the grid in Fig. 16.

	b	d
b	odd	even
d		odd

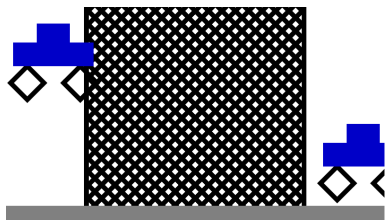


Figure 16: Drive.

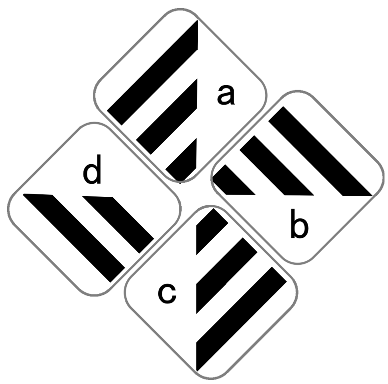


Figure 17: Pinwheel.

the clock. When the angular width of the object is an even multiple of the angular width of the stripes, the footsteps illusion occurs most strongly. When the angular width of the object is an odd multiple of the angular width of the stripes, the inchworm illusion occurs. The design of this clock is currently patent pending.

Table 8: Pattern of the widths of and distances between objects a and c in Fig. 17 with respect to the grid in Fig. 18.

	a	c
a	odd	even
c		odd

Table 9: Pattern of the widths of and distances between objects b and d in Fig. 17 with respect to the grid in Fig. 18.

	b	d
b	odd	even
d		odd

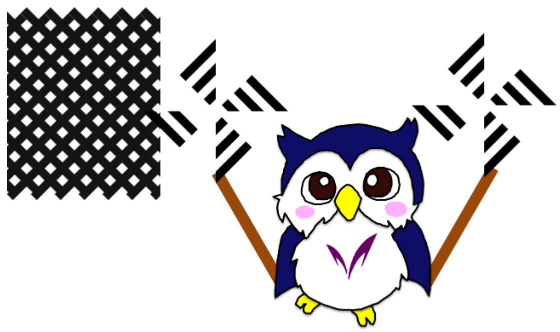


Figure 18: Pinwheels.

5.2 Signboard

Another example of an application is an optical illusion signboard. In this case, the methodology is slightly different from that of the previously discussed examples. Both the footsteps illusion and the inchworms illusion have three factors: the object, the stripes, and the viewpoint. Until now, we have discussed only the case where the object or the stripes move. Next, we discuss the case where the viewpoint moves. In order to achieve the relative motion between the object and the stripes when the viewpoint moves, we have to place the object and the stripe at different distances from the viewpoint. For this purpose, we create a slit pattern by replacing the white stripes with holes. Then we can observe

Table 10: Pattern of vertical widths of and distance between objects in Fig. 19 with respect to the grid in Fig. 20.

	a	c
a	odd	even
c		odd

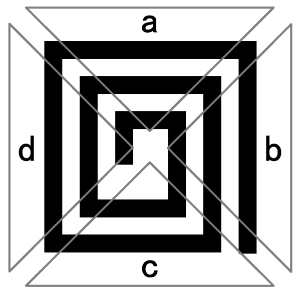


Figure 19: Eddy.

Table 11: Pattern of horizontal widths of and distance between objects in Fig. 19 with respect to the grid in Fig. 20.

	b	d
b	odd	even
d		odd

the object through the slits. The procedure for creating such a signboard is as follows.

As shown in Fig. 22, we define ω_0 as the width of a slit (also, the width of the wall segments separating neighboring slits), k as the ratio of the distance between the object and the slit to the distance between the slit and the line along which the viewpoint moves, ω as the width of the projected image of the slit on the object plane, L as the width of the object, and F as length of both the interval in which the object clearly moves, as well as that in which it appears to have stopped, in the basic footstep illusion. Suppose that ω_0 and k are given. Then we can calculate ω, L, and F in the following way.

Let A be the viewpoint, D and E be the edges of a slit, and B and C be the points of intersection of the half lines from A through D and E and the object plane, as shown in Fig. 23. We examine the triangles ABC and ADE. The two triangles are similar because $\angle A$ is a common angle and $\angle ABC$ and $\angle ADE$ are corresponding angles. Therefore, we derive

$$\omega_0 : \omega = k : (k+1),$$

and so

$$\omega = \frac{\omega_0(k+1)}{k}. \tag{3}$$

Next we calculate L. The value of L depends on the desired apparent motion. For the footsteps illusion, L should be an even multiple of ω. For the inchworm

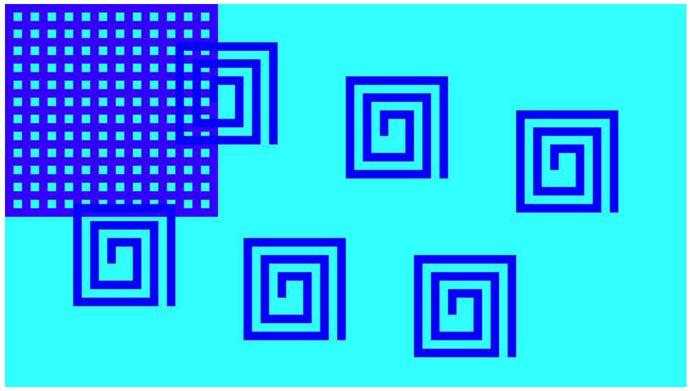


Figure 20: Eddy Current.

illusion, L should be an odd multiple of ω [14]. Therefore, we calculate L as

$$L = \begin{cases} 2n\omega & \text{(In the case of the footsteps illusion)} \\ (2n+1)\omega & \text{(In the case of the inchworm illusion)} \end{cases} \quad (n \in \mathbf{N}). \tag{4}$$

We substitute eq. (3) into eq. (4), and we get

$$L = \begin{cases} \frac{2n\omega_0(k+1)}{k} & \text{(In the case of the footsteps illusion)} \\ \frac{(2n+1)\omega_0(k+1)}{k} & \text{(In the case of the inchworm illusion)} \end{cases} \quad (n \in \mathbf{N}). \tag{5}$$

Next we calculate F. Let us define A, B, C, D, and E as shown in Fig. 24. The two triangles ABC and ADE are similar because $\angle A$ is a common angle and $\angle ABC$ and $\angle ADE$ are corresponding angles. Therefore, we get

$$F : \omega_0 = (k+1) : 1, \tag{6}$$

and so

$$F = \omega_0(k+1). \tag{7}$$

In this way, we can make an optical illusion signboard, shown in Fig. 25, which realizes the same optical illusion art as “For Egg Laying” shown in Fig. 11. This signboard is designed such that ω_0 is 8 mm, the distance from the slit to the signboard is 70 mm, and k is 50. These parameter values imply that the illusion can be expected to occur most strongly when seen at a distance of 3500 mm, which is consistent with our actual observations.

6 Conclusion

In this paper, first we determine conditions under which the strength of the footsteps illusion is maximized, which are when the object width is an even

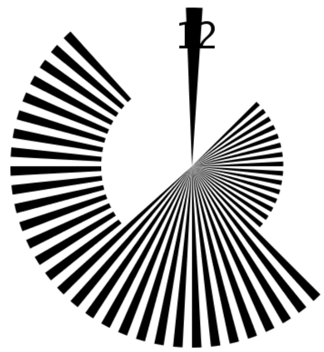


Figure 21: Optical illusion clock.

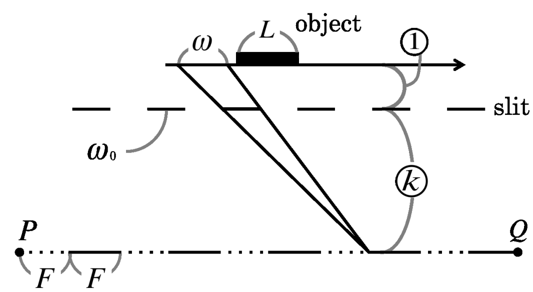


Figure 22: Basic structure of the signboard. The viewpoint moves from P to Q.

multiple of the stripe width, and those under which the effectiveness of the inchworm illusion is maximized, which are when the object width is an odd multiple of the stripe width.

Next, we classified the apparent motions into eight patterns according to the widths of a pair of objects and the distance between them, and created new illusion artworks by combining these eight patterns. Here, we introduced six artworks, but anyone can create illusion artworks freely by using these ideas. Other artworks can be seen at the first author's web page [15].

Finally, we showed applications to a clock and a signboard.

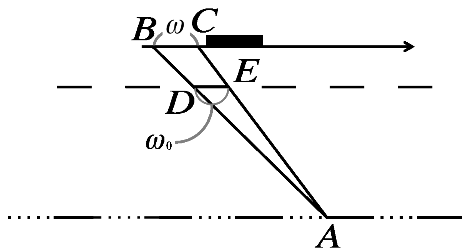


Figure 23: Relationship between the stripe width ω_0 and the effective width ω.

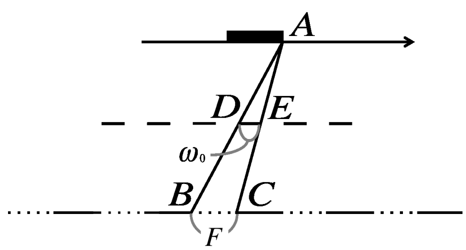


Figure 24: Relationship between the stripe width ω_0 and the effective width F.

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References

[1] M. Anno, Fushiginae (in Japanese), Fukuinkansyoten, Tokyo, Japan, 1971.

[2] S. Anstis, Footsteps and inchworms, *Perception*, 30, 785-794 (2001).

[3] S. Anstis, Moving objects appear to slow down at low contrasts, *Neural Networks*, 16, 933–938 (2003).

[4] S. Anstis, Factors affecting footsteps: contrast can change the apparent speed, amplitude and direction of motion, *Vision Research*, 44, 2171–2178 (2004).

[5] F. H. Bool, J. R. Kist, J. L. Locher, and F. Wierda, *M. C. Escher: His Life and Complete Graphic Work with a Fully Illustrated Catalogue*, Harry N. Abrams, Inc., New York, 1992.

Figure 25: Signboard.

[6] S. Dali, DalihaDalida Dalichosakusyu (in Japanese), *Michitan*, 2011.

[7] S. Fukuda, Fukudashigeo Sakuhinsyu (in Japanese), Koudansya, Tokyo, Japan, 1979.

[8] W. Kriegeskorte, Arcimboldo, *Taschen Basic Art*, Taschen America Llc, Köln ,2004.

[9] T. Goto and H. Tanaka, Sakushi no Kagaku Handobukku (in Japanese), *University of Tokyo Press*, Tokyo, Japan, 2005.

[10] P. D. L. Howe, P. G. Thompson, S. M. Anstis, H. Sagreiya, and M. S. Livingstone, Explaining the footsteps, belly dancer, Wenceslas, and kickback illusions, *Journal of Vision*, 6 (12), 1396–1405 (2006).

[11] A. Kitaoka, Sakushi nyumon (in Japanese), *Asakurasyoten*, Tokyo, Japan, 2010.

[12] A. Kitaoka, Torikkuato zukan Fushigie (in Japanese), *Akanesyobou*, Tokyo, Japan, 2011.

[13] K. Miura, A. Osa, and H. Miike, A simulation of the footsteps illusion using a reaction diffusion model, *IEEJ Transactions on Electronics, Information and Systems, A publication of Electronics, Information and System Society*, 129, 1156–1161 (2009).

[14] J. Ono, A. Tomoeda, and K. Sugihara, Footstep illusion art, *NICOGRAPH 2012*, 25–26 (2012).

[15] http://www.kisc.meiji.ac.jp/∼cs21001/

[16] H. Ōuchi, *Japanese Optical and Geometrical Art: 746 Copyright-Free Designs for Artists and Craftsmen*, Dover Publications, Inc., New York, 1977.

[17] A. Seckel, Masters of Deception: Escher, Dali & the Artists of Optical Illusion, *Sterling Pub Co Inc*, 2008.

[18] N. Washino and A. Ishihara, Footsteps, *Forum on Information Technology*, 9, 531–532 (2010).